

**Turning Routine Exercises into
Activities that Teach Inquiry:
Examples from Discrete Mathematics**

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Floor & Ceiling Activity



Partners
(Pair)

+ Spies & Ambassadors

Turn to a neighbor & quickly introduce yourself.

Complete activity on handout, help/check partner.

Done or stuck? Each go to a different pair, then return.

3-5 min

1 Floor and Ceiling Activity

The floor and ceiling functions take a real number input and find a nearby integer to output.

Definition 1.1 (Floor and Ceiling).

- (a) For a real number x , the **floor** (or **greatest integer**) of x , denoted $\lfloor x \rfloor$, is the result of rounding x down to the nearest integer less than or equal to x . Officially, $\lfloor x \rfloor =$ the greatest (largest) integer k such that $k \leq x$. For example, $\lfloor 2.4 \rfloor = 2$ and $\lfloor -2.4 \rfloor = -3$. This name comes from the fact that we look down to see the floor of a room. Notice the floor is the first integer at or to the left of the number on the real number line.

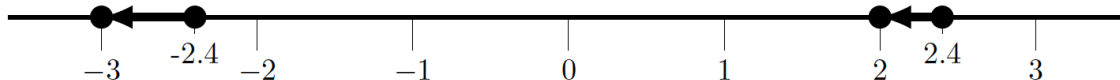


Figure 1.1: Calculating the floor by looking left on the real number line.

- (b) For a real number x , the **ceiling** of x , denoted $\lceil x \rceil$, is the result of rounding x up to the nearest integer greater than or equal to x . Officially, $\lceil x \rceil =$ the least (smallest) integer k such that $k \geq x$. For example, $\lceil 2.4 \rceil = 3$ and $\lceil -2.4 \rceil = -2$. This name comes from the fact that we look up to see the ceiling of a room. Notice the ceiling is the first integer at or to the right of the number on the real number line.

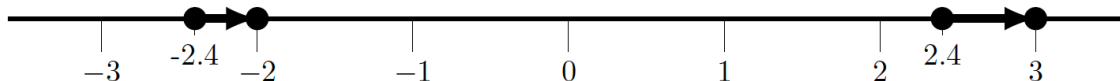


Figure 1.2: Calculating the ceiling by looking right on the real number line.

- (c) Floor and ceiling are at the same level as parentheses in the order of operations.

Back of hand

You try working with floor and ceiling

Activity 1.2 (Floor and Ceiling).

(a) Evaluate each of the following quantities.

$$\lfloor -0.32 \rfloor \quad \lfloor 52 \rfloor \quad \lceil -0.32 \rceil \quad \lceil \frac{80}{3} \rceil \quad \lfloor \lceil 103.4 \rceil \rfloor \quad \lceil 9.5 \rceil - \lfloor 9.5 \rfloor \quad \lceil .7 + .5 \rceil$$

(b) If expect 40 students to attend a talk. If I want to buy enough cans of sparkling water for every student to get one, then how many 12-packs of cans should I buy? What if there are n students? Your answer should involve n and floor or ceiling.

(c) When is $\lfloor x \rfloor = \lceil x \rceil$ for a real number x ?

(d) Give an example of a real number x that is not an integer where $\lceil 2x \rceil = 2\lceil x \rceil$.

(e) Give a counterexample to the conjecture that $\lceil 2x \rceil = 2\lceil x \rceil$ for all real numbers x .

(f) If $\lfloor x \rfloor = 6$, what can we say about $\lceil x \rceil$?

(g) If x is a real number such that $\lfloor x \rfloor \neq \lceil x \rceil$, then how are $\lfloor x \rfloor$ and $\lceil x \rceil$ related? Be specific.

(h) The function $r : \mathbb{R} \rightarrow \mathbb{R}$ is defined by $r(x) = \lfloor x + 0.5 \rfloor$. Describe the effect of r in words.

(i) How can $\lfloor -x \rfloor$ can be simplified to an expression with the negative sign on the outside?

Inquiry-based activities are

Interesting: cool math, relevant to students

Open-ended/open-middle: multiple answers or possible strategies.

Scaffolded: low-floor to high ceiling.

easy-medium-sideways-challenging

Exploratory: questions, conjectures, counterexamples

How does the activity (handout) we did meet these criteria?



Neighbor
(Pair)

3-5 min

Read more about it . . .

**Turning Routine Exercises into Activities
That Teach Inquiry: A Practical Guide**

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Free to MAA members.



**Up for class-testing OER
discrete text in Fall 2025?**

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